3.8 Local Linear Approximation

(Cf. Barron's 3/31)

* semi-important topic on AP Exam
* main idea: use tangent line to approximate function

Jan 11-7:15 AM

Eqn of a line (point-slope format)

A line with slope m going through the point (a, b) (or (a, f(a))) has eqn.

AN-EE LION

$$y-b=m(x-a)$$

this comes from slope formula

$$m=\frac{y-b}{x-a} \quad \frac{(\text{diff in } y)}{(\text{diff in } x)}$$

2. What is different in calculus?

Different: when I get m from

$m = f'(a)$

(where the pt is (a,b))

[gives eqn of tangent line]

3.8 Local Linear Approx.

takes:

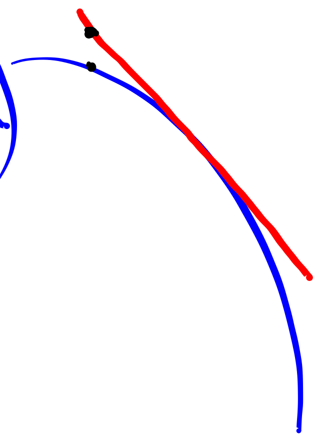
$$y - f(a) = f'(a)(x-a)$$

3.8 version:

$$y = f(a) + f'(a)(x-a)$$

y of the line

$$f(x) \approx f(a) + f'(a)(x-a)$$



of the curve

Jan 11-7:15 AM

Approximate $\sqrt{17}$ without peeking

$$\sqrt{17} = \sqrt{16+1}$$

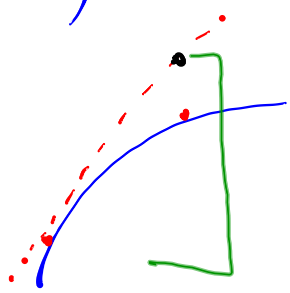
$$f(x) = \sqrt{x} = x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

$$f'(16) = \frac{1}{2\sqrt{16}} = \frac{1}{2\cdot 4} = \frac{1}{8}$$

$$f(x) = \sqrt{17} \approx \underline{f(a)} + \underline{f'(a)}(x-a)$$

$$a = 16$$



$$\sqrt{17} \approx 4 + \frac{1}{8}(17-16)$$

$$= 4 + \frac{1}{8}(1) = \underline{4.125}$$

4.123

Jan 11-7:48 AM

estimate $\sqrt{15}$

$a=16$

$\sqrt{15} = \sqrt{16-1}$

$f(x) = \sqrt{15} \approx f(a) + f'(a)(x-a)$

$= f(16) + f'(16)(15-16)$

$= 4 + \left(\frac{1}{8}\right)(-1)$

$= 4 - \frac{1}{8} = 3.875$

Actual Value: 3.872

Use Local Linear Approximation to approximate sin(x) near $x=0$

$$\star\ f(x) \approx f(a) + f'(a)(x-a)$$

or

$$f(a+h) \approx f(a) + f'(a)(h)$$

$f(x) = \sin x$

$f'(x) = \cos x$

$f'(0) = \cos 0 = 1$

$f(x) \approx \sin 0 + 1(x-0)$

or

$f(x) = \sin(x) \approx x$

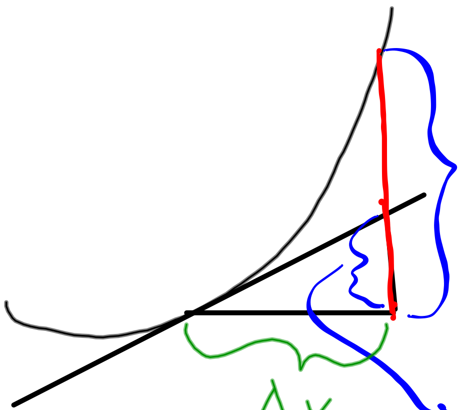
Jan 11-8:08 AM

differentials

dy is a differential of y

dx is a differential of x

The picture we think of



Δy = chg in y on actual curve

dy = the infinitesimally small change in y as we take the limit

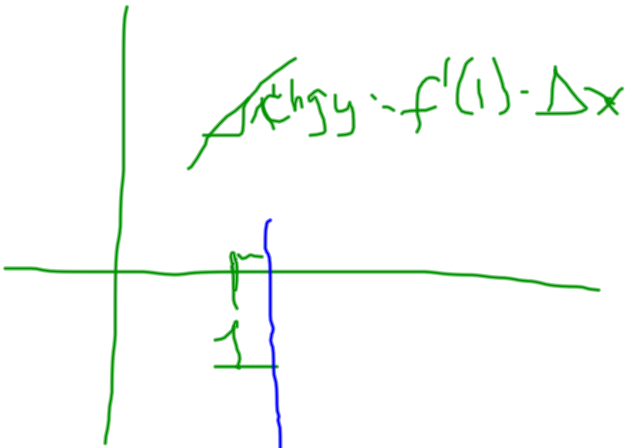
Δx or dx

$\frac{\Delta y}{\Delta x}$ = not slope, but an actual ratio

$\frac{dy}{dx} = f'(x)$

HW: 3.8 / 5-11, 19-23, 33-37 } odd

$$f(a+\Delta x) \approx f(a) + f'(a)(\Delta x)$$



(x−a) distance from a to my unknown x

x close to 1 = 1+Δx

$$\sim m(x-a) = \sim m(x-1) = \sim + m(1+\Delta x - 1) = \sim + m(\Delta x)$$

$$m = f'(1)$$

3.8/5 use local lin. ap. close to $x=1$

$f(x)=x^{15}$ $f'(x)=15x^{14}$ ...

$(1+h)^{15}$

$$f(x)\approx f(a)+f'(a)(x-a)$$

$a=1$

$$f(r)\approx f(1)+f'(1)(r-1)$$

$$=1^{15}+15(r-1)$$

r is unknown but close to 1

$r=1+h$

h close to 0

$$f(1+\cancel{h})\approx 1+15((1+h)-1)=$$

$$1+15(\cancel{h})$$

Jan 11-9:05 AM

9) $f(x)=x^4$ $\quad f'(x)=4x^3$ $\quad f'(1)=4$

$$f(x)\approx f(a)+f'(a)(x-a)$$

$a=1$

$$f(x)\approx f(1)+f'(1)(x-1)$$

$$f(x)\approx 1+4(x-1)$$

$\Delta x = x-1$

$(\Delta x+1)=x$

$$f(1+\Delta x)\approx 1+4(\Delta x)$$

S.B's

4eva!

Jan 11-11:22 AM

Jan 11-11:22 AM